LECO Laboratory

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What is Control Engineering?

- A discipline that applies control theory to design systems that regulate (control) the behavior of other systems
- Uses feedback from sensors to monitor system performance and apply corrective control actions

What does LECO do?

- Use system models to compute efficient control feedback systems, to be verified with simulation and refined during live deployment
  - Systems used for experimentation include the Quanser Quad-Tank and the Quanser 2D-Copter
- Advanced algorithm development and verification

Model Predictive Control (MPC)

- MPC is an advanced control strategy that accounts for pre-defined constraints, making it uniquely suited for multi-input-multi-output (MIMO) systems
    - It is an iterative process that optimizes state predictions and control actions for a limited horizon
- MPC relies on dynamic models of a given system that are usually obtained through system identification
- The general form of a MPC optimization problem is shown below

$$J = \frac{1}{2}\left(x_N^T P_N x_N + \sum_{t=k+1}^{k+N} x_t^T Q_t x_t + \sum_{t=k}^{k+Nu-1} u_t^T R_t u_t\right)$$

$$\text{Subject to: } x_{t+k+1} = Ax_{t+k} + Bu_{t+k}$$
$$for\ k = 0,1, Np - 1\ ;$$
$$x \in \mathbb{X}, u \in \mathbb{U}$$

Solving MPC as a Linear Complementarity Problem

- The MPC formulation previously shown is able to be simplified using linear algebra first into a quadratic program (QP) and then into a linear Complementarity Problem (LCP) using linear algebra. The resulting problem statement is shown below.

$$w = Mz + q$$
$$\textit{subject to: } z \geq 0 \, ; \;\; z^T(Mz + q) = 0$$

- This linear optimization can be solved using a number of algorithms, namely the Lemke Pivoting Algorithm. This algorithm introduces a ‘covering vector’ which is used to initially drive a certain variable to zero, revealing a solution to the LCP. This vector is almost always a column of ones equal to the dimensions of matrix M
- This solution can then be converted back into MPC form to give a final answer to the MPC optimization

Developing Pivot Based Algorithms

- Using the theory discussed, a quadratic program was developed using MatLab that utilized Lemke's Algorithm to solve the resulting LCP. The result and speed of this algorithm, QPSolver, was compared with other well known and available solvers. The solvers compared are MatLab's quadprog and mpcActiveSetSolver. The computation time results of the experiment are shown below.

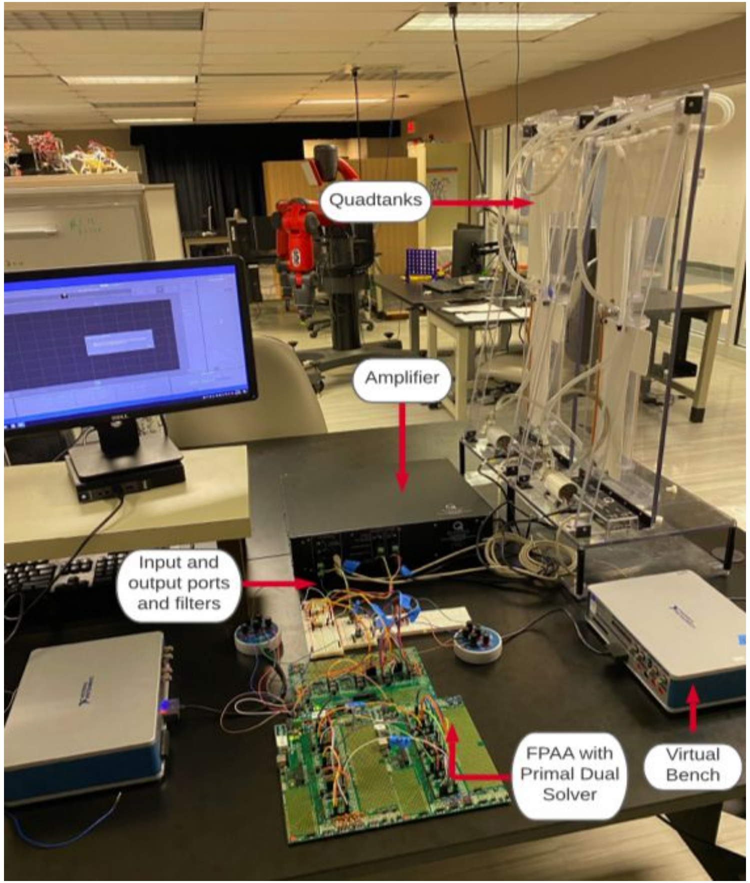
	Computation Time (ms)
QPSolver	0.1445
Quadprog	5.74
mpcActiveSetSolver	0.4281

Current and Future Work

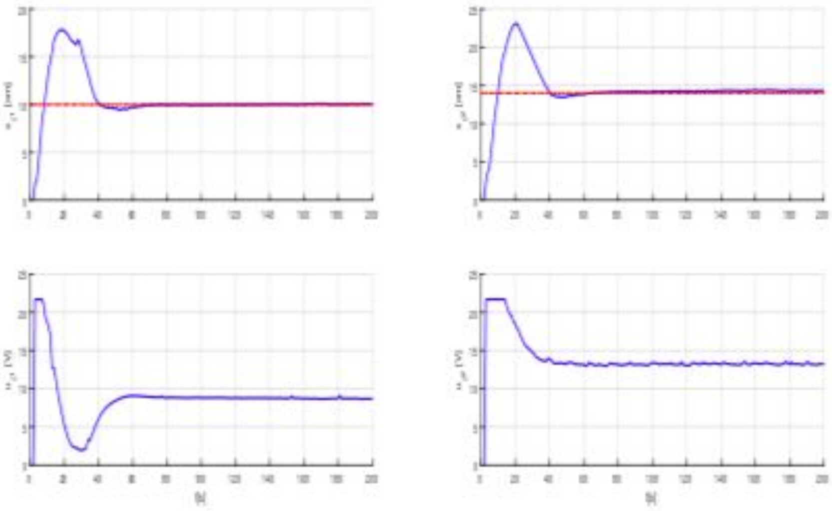
- Based on the results of the computation time test, it is reasonable to conclude that Lemke's Algorithm performs better than general solvers.
- Currently, a n-step routine is being implemented to complement the Lemke Algorithm in an attempt to prove that is allows the MPC problem to be solved in less iterations than the Lemke algorithm on its own
- The n-step routine inputs the matrix M and vector q from the LCP formulation and computes a vector p that when used as the covering vector results in the Lemke Algorithm solving the LCP faster

Analog MPC

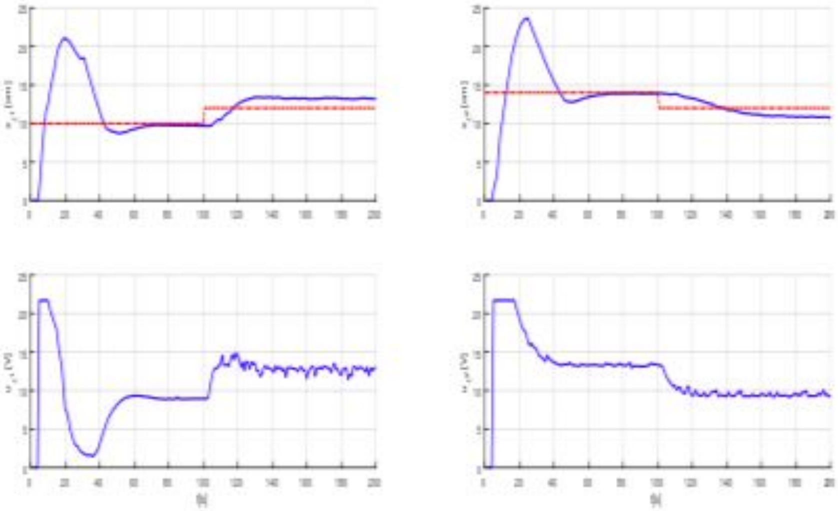
- Analog MPC on the quad tank system.
- Used Anadigm FPAA
- Used FPAA to control tanks to certain setpoints
- Comprised of a solver and an observer
- Observer estimates states that cannot be read by sensors



Analog MPC Results



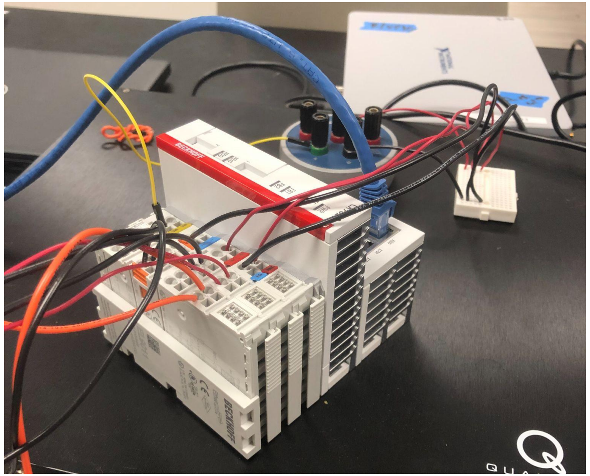
Step at t = 0



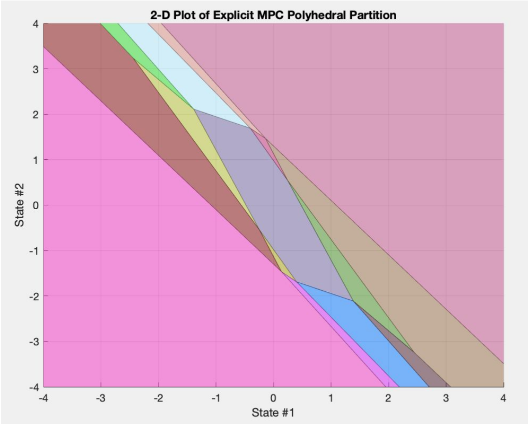
In-Phase Step at t = 0
Out-of-Phase Step at t = 100

Explicit MPC on a Programmable Logic Controller (PLC)

- What is a PLC?
  - Robust, industrial digital computer used for process control. Generally simple tasks like switching, or PID control

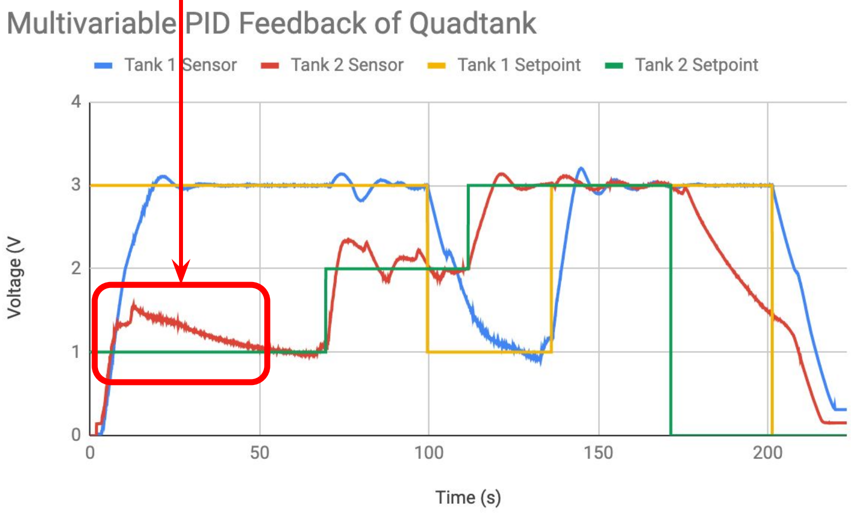


- What is Explicit MPC?
  - An extension of traditional MPC, that uses pre-computed solutions based on states at given a sample time rather than perform an online optimization problem



Explicit MPC on a Programmable Logic Controller (PLC)

- Why?
  - PLCs are an industry standard with an existing community of trained technicians and familiarity
  - It would be desirable to use this inexpensive and readily available technology for more complex and efficient control
    - Improve manufacturing process control for applications such as robotic arms, flow control, or assembly lines with an adaptable solution



LECO Laboratory Achievements 2020-2021

- Publication accepted at American Control Conference entitled “Analog Solver for Embedded Model Predictive Control with Application to Quadruple Tank System”
  - Paper focused on implementation of analog MPC on FPAA
- Control Conference Africa (CCA) paper currently being written on the topic of implementing an n-step vector into the Lemke pivot algorithm to solve MPC problems more efficiently
  - Rudimentary experimentation has shown promise that this effectively limits the number of pivot interactions required to find a solution