An Analysis of the Level Ancestor Problem

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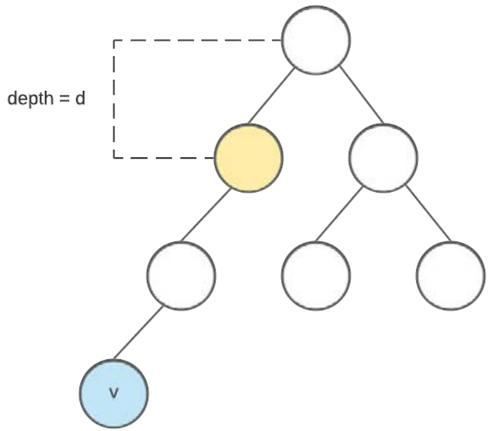
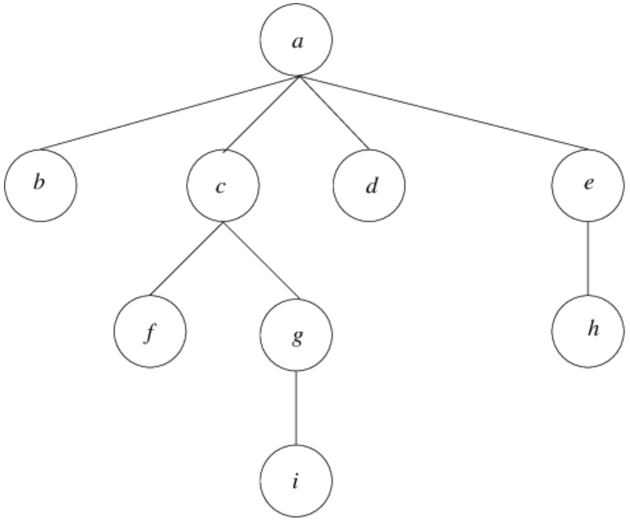
The Level Ancestor Problem

- In a tree T with n nodes, answer a query of the form LA(v, d)
  - Query aims to find the ancestor of the node *v* that is at a depth of *d* from the root
  - This can be done is O(n) time by tracing the path from the target node to the root until reaching depth *d*, but it is impractical for larger trees
- Preprocessing for the algorithms aims to store the tree in data structures so that queries can be completed more efficiently



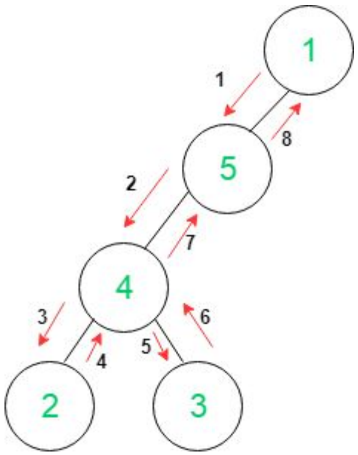
Applications

- Level ancestor implementations can be used to aid in many other algorithms
  - Least Common Ancestor
    - Used in string processing, computational biology, and complex distributed systems, among many other applications
  - Space-Efficient Ordinal Trees
    - XML document representation for XPath queries
  - Range-Aggregate Queries on trees

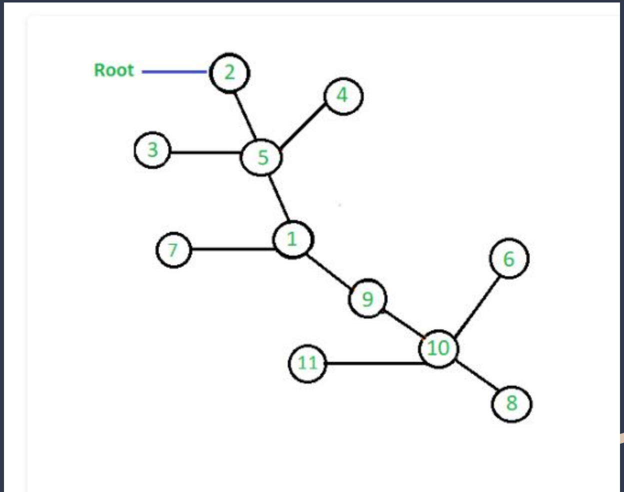


Our Project

- We aimed to code (in C) the algorithms outlined in different papers and compare the experimental time for tree preprocessing and querying
- We examined eight algorithms for static binary trees
- Random Tree Generation:
  - Trees were represented in Euler format that follows a Depth-First Search preorder
    - Binary Representation:
      - 1 represents moving down in the tree
      - 0 represents moving upwards in the tree
      - Example: 11101000
        - Euler Representation= 154243451
  - Generated via split-tree method
    - Creates trees where both the average node depth and tree depth are logarithmically bound
    - Allows for easy construction of Euler representation and skewed trees

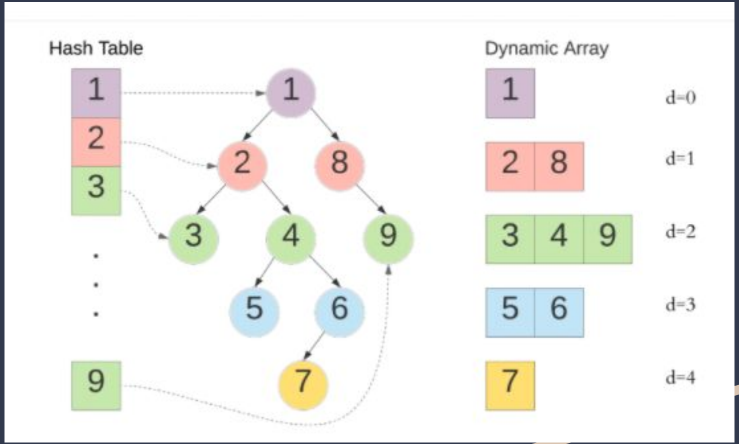


The Algorithms



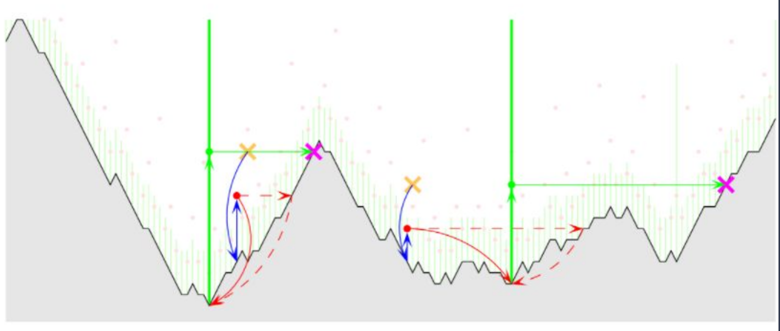
- Table Algorithm:
  - Preprocessing = $O(n^2)$, Query = O(1)
  - Uses a 2D-array to store the list of all ancestors of each node derived by a traversal in DFS order
- Jump-Pointer Algorithm:
  - Preprocessing = O(nlogn), Query = O(logn)
  - Uses pointers from each node to its ancestors, allowing for the distance to the ancestor at depth *d* to be traversed in jumps of at least half the remaining distance to the ancestor
- Ladder Algorithm:
  - Preprocessing = O(n), Query = O(logn)
  - Perform a longest path decomposition on the tree and extends the paths to reach the root so that a node at height *h* can be queried for ancestors of at least height 2*h*
- Jump Ladder:
  - Preprocessing = O(nlogn), Query = O(1)
  - Combines the Jump-Pointer and Ladder Algorithms
  - First jumps up the tree, then uses the ladder to move up towards the ancestor node

The Algorithms



- Macro-Micro Algorithm:
  - Preprocessing = O(n), Query = O(1)
  - Uses the other four algorithms
  - Nodes are divided into macro and micro subtrees based on their weight
  - Macro nodes have use of the jump-pointer method
  - Micro trees use table algorithm
- Menghani-Matani (Google-Facebook) Algorithm:
  - Preprocessing = O(n), Query = O(logn)
  - Perform a pre-order traversal of the tree where each node is given a integer tag starting at 1 for the root
  - Each node, in increasing tag value, is stored in a specific dynamic array depending on its depth
  - Find the largest node label less than the label of node *v* in the array for depth *d* and return the pointer for that label

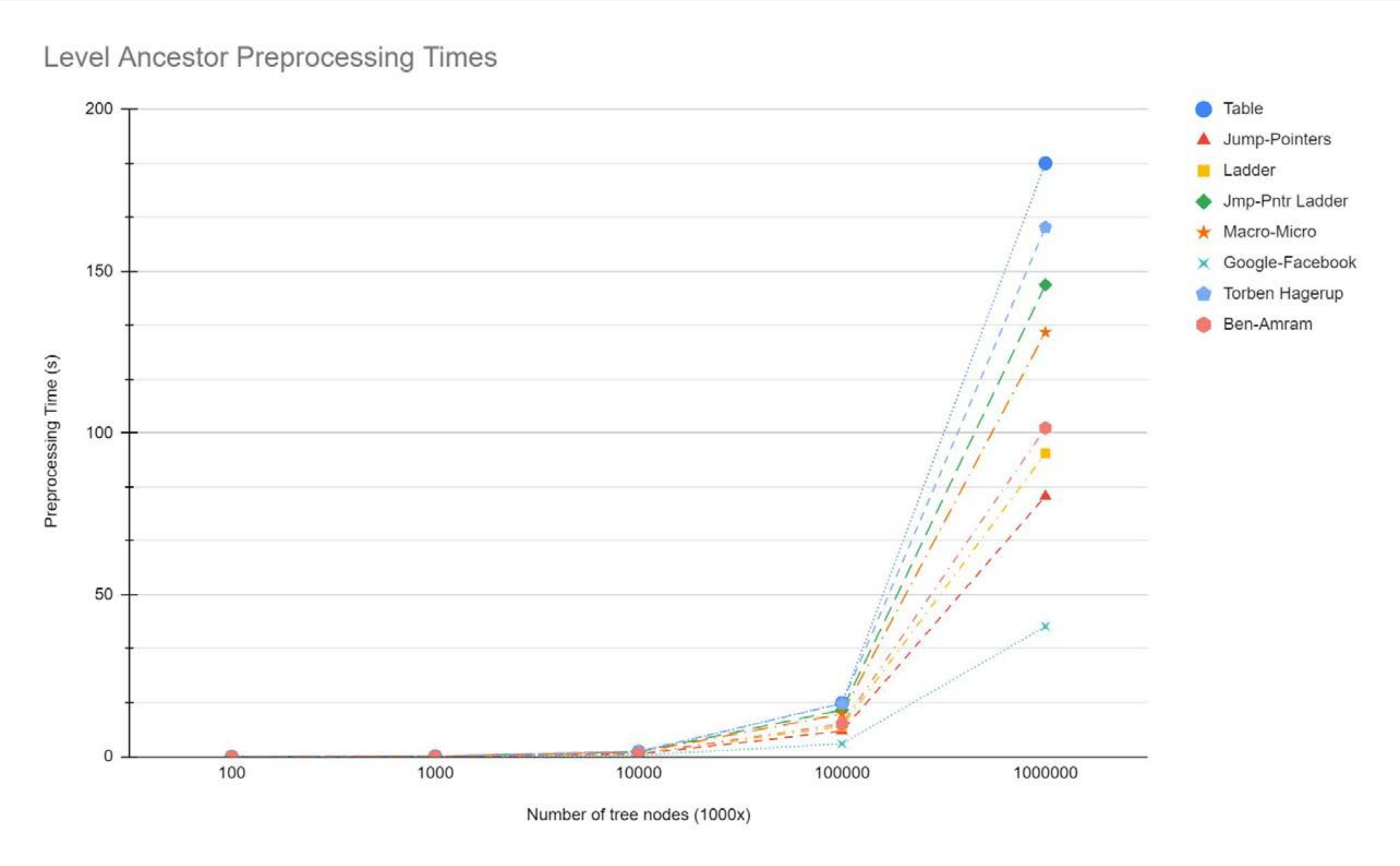
The Algorithms

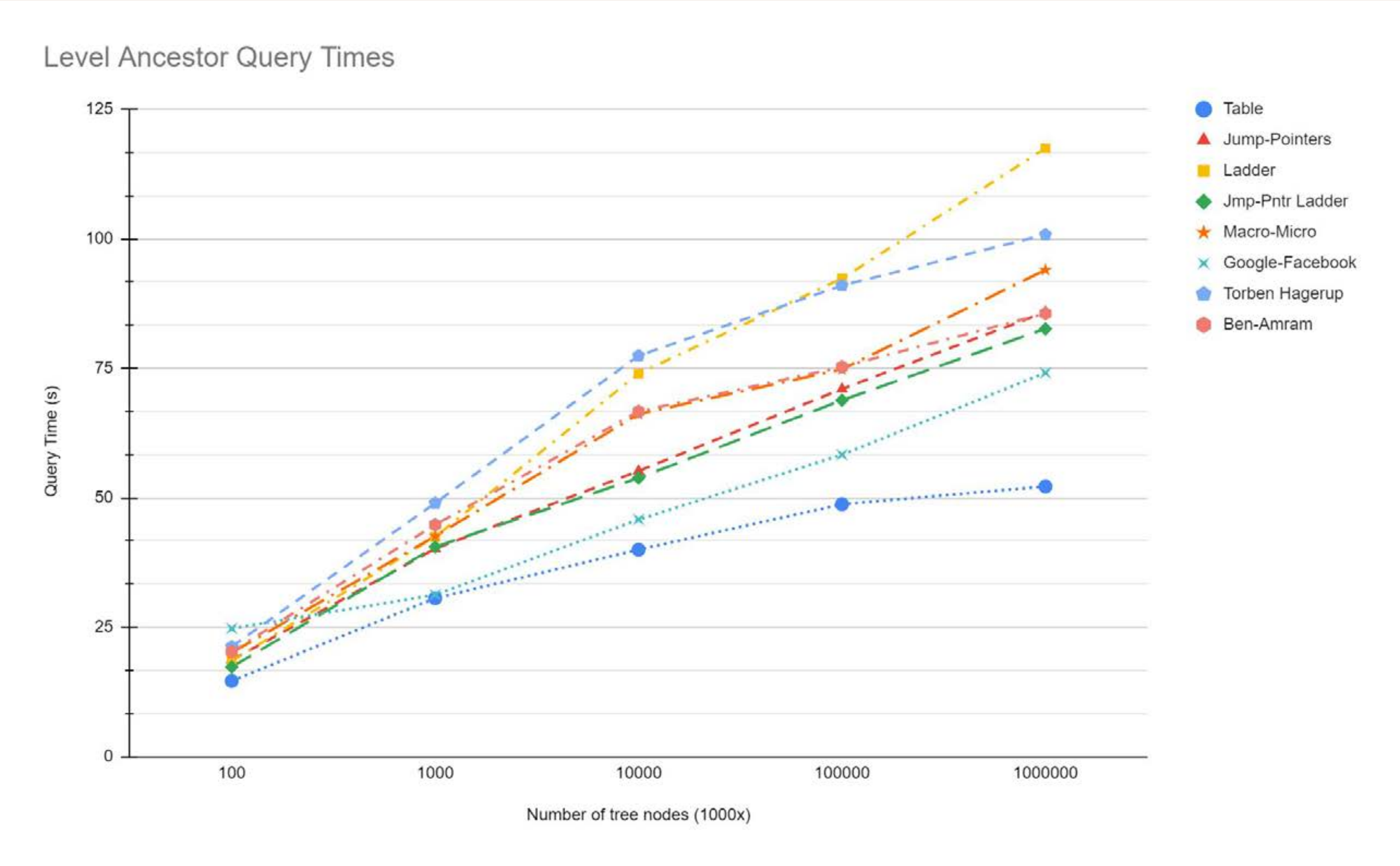


- Torben Hagerup:
  - Preprocessing = O(n), Query = O(1)
  - Uses the find-smaller method to create a 2D structure of peaks and valleys
  - Finds the deepest valley nearest to the node *v* and then uses the ladder method until reaching the height of the given depth *d*
  - Then the node to the right of this height in the 2D structure is selected
- Ben-Amran:
  - Preprocessing = O(nlogn), Query = O(n)
  - Uses the find-smaller methods which takes an array *A*, an index *x*, and an integer *y*
  - Finds an index $u > x$ such that $A_u \leq y$

Results

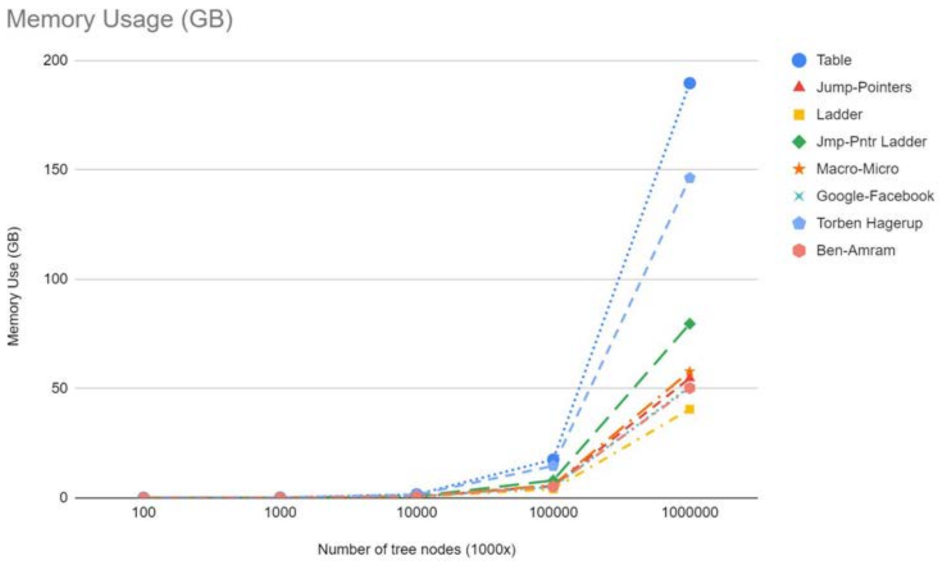
- Multiple trials were run with varying numbers of nodes from 100,000 to 1 billion and 100 million queries per tree
- Tests were run on the TCNJ cluster as submitted job scripts to an AMD EPYC2 processor with 512 GB of memory
- Preprocessing:
  - Fastest - Google-Facebook Algorithm
  - Slowest - Table Algorithm
- Querying:
  - Fastest - Table Algorithm
  - Slowest - Torben Hagerup & Ladder Algorithms


Level Ancestor Preprocessing Times
Preprocessing Time (s)
200
150
100
50
0
100
1000
10000
100000
1000000
Number of tree nodes (1000x)
Table
Jump-Pointers
Ladder
Jmp-Pntr Ladder
Macro-Micro
Google-Facebook
Torben Hagerup
Ben-Amram


Level Ancestor Query Times
Query Time (s)
125
100
75
50
25
0
100
1000
10000
100000
1000000
Number of tree nodes (1000x)
Table
Jump-Pointers
Ladder
Jmp-Pntr Ladder
Macro-Micro
Google-Facebook
Torben Hagerup
Ben-Amram

Results

- ➢ In addition to time taken, we also examined memory usage for each algorithm
  - ○ As expected, the Table Algorithm uses up the most memory space



Future Research

- Examine ways to improve upon the existing algorithms
  - Could incorporate more assembly code that speeds up processing time
- Incorporate dynamic algorithms that allow for adding leaves to trees
  - Alstrup & Holm Algorithm
    - Implements an improved version of the macro-micro algorithm
  - Dietz Algorithm
  - Adjust static algorithms to allow for dynamic trees

References

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